Marketing Percolation

J. Goldenberg[1], B. Libai[2], S. Solomon[3], N. Jan[4], and D. Stauffer[5]

[1] School of Business and Administration, Mt. Scopus, Hebrew University, Jerusalem 91905, Israel
[2] Davidson Faculty of Industrial Engineering and Management, Technion, Haifa 32000, Israel.
[3] Racah Institute of Physics, Givat Ram, Hebrew University, Jerusalem 91904, Israel
[4] Physics Department, St Francis Xavier University, Antigonish, Nova Scotia, Canada B2G 2W5
[5] Institute for Theoretical Physics, Cologne University, D-50923 Köln, Germany
e-mail: sorin@vms.huji.ac.il

Abstract: A percolation model is presented, with computer simulations for illustrations, to show how the sales of a new product may penetrate the consumer market. We review the traditional approach in the marketing literature, which is based on differential or difference equations similar to the logistic equation (Bass 1969). This mean field approach is contrasted with the discrete percolation on a lattice, with simulations of "social percolation" (Solomon et al 2000) in two to five dimensions giving power laws instead of exponential growth, and strong fluctuations right at the percolation threshold.

1. Introduction.

If the amount of activity in an academic area reflects its importance, then research on the diffusion of innovations, with over 4,000 diffusion publications since 1940, is one of the most important areas in the social sciences. "No other field of behavioral science research represents more effort by more scholars in more disciplines in more nations" [1]. Marketing's considerable share of the output in this research stream reflects not only the importance of new products, but also the role of diffusion research in helping managers to better plan their entry strategy, target the right consumer and anticipate demand so as to have an efficient and effective promotion, production and distribution strategy. (We use here the terminology of marketing theory and call them diffusion models, whereas the physics of diffusion is quite a different process.) Diffusion as marketing experts define it is the development (increase) of sales over time (not spatial again), it is viewed as analogous to epidemics with increasing number of ill people. So is the new product: increasing number of adopters is in essence the diffusion process. The growth of new products is a complex process which typically consists of a large body of consumers interacting with each other over a long period of time.

Distressingly, often only aggregate data on adoption (i.e. the sum of all previous sales etc.) is available to researchers for analysis, as is generally the case with market level diffusion models [2,3]. (Aggregate data means that the sales are measured once in a quarter or a year, without any attention to spatial distribution, and no attention to the individual buyer.) Even when collecting data at the individual level, diffusion research surveys consist of correlated data gathered in one "snap shot" survey of consumers, a methodology that

amounts to freezing the diffusion process, making the continuous time-dependent process timeless [1].

Hence, it is not surprising that much of the theoretical base to the diffusion of innovations is grounded on repeatedly analyzed small number of data sets, in which researchers could actually follow the diffusion process within small social systems, such as the cases of the diffusion of hybrid corn among farmers in Iowa [4], antibiotics among US physicians [5] or family planning in Korean villages [6]. While the impressive contribution of these studies is evident, new tools should be considered to analyze the fast changing and complex environment of new product growth.

The small set of available individual based data poses another research dilemma: The small number of cases can not offer us an over-view of how collective behavior emerges from changes in individual characteristics. The span of individual level parameters is too small to allow for developing an explanation of their relations to the diffusion parameters or to predict them from the diffusion parameters.

Thus, the modeling of the diffusion of new products lies between two extremes.

Aggregate, or market level, diffusion models, such as the Bass model [7], an equation similar to what physicists call the logistic equation or Verhulst factor, are based on market level data and assume a large degree of homogeneity in the population of adopters. Basically, the diffusion of innovations models primary premise is based on the assumption that communication between individuals, is central to the new product's growth.

One of the advantages of diffusion models is that they provide a relatively easy and parsimonious analytic way to look at the whole market and interpret its behavior, yet, still based on rich and empirically based theory. Another advantage is that very often the market level is also the level managers will be mostly interested in. Finally, aggregate models can be estimated with market level data such as number of adoptions in a given year or average price, which are relatively easy to get.

This simplicity is also associated with some critique on the aggregate approach to diffusion. One shortcoming is that the models make strong and simplifying assumptions on the behavior of individuals, for example the lack of heterogeneity among adopters. Also, the ability to test the assumptions these models make with very limited data at the aggregate level can be questioned [8].

Individual level models, on the other hand, acknowledge differences between consumers (e.g., difference in utility among potential adopters and their affect on adoption). Generally they follow economic theories (e.g. [9]) and assume that individuals maximize some personal objective function such as utility of the product, and may update their beliefs as more information arrives at the market. Thus, individual level models can be viewed as more behaviorally based than aggregate models.

Aiming at explaining aggregate adoptions in the market level, restrictions on the heterogeneity in behavior among individuals are sometimes introduced, and individual levels models are aggregated to provide an explicit diffusion function at the market level (e.g., [10]). Yet, the use of market level data to calibrate individual level models is still not very common, partly because the very limited aggregate level data do not really allow individual level testing, as the case in the traditional diffusion models.

Our study synthesizes individual and aggregate level modeling in a way which may

help to overcome some of the outlined barriers. We follow diffusion theory and its emphasis on the communication behavior as a driver of new product growth, and generate a variety of possible dynamics to explore their influence on the aggregate level. Percolation enables us to perform sensitivity analysis and examine the effect of changes in the parameters in the individual level on the aggregate level, and thus overcome some of the limitations that follow the use of few data points at the aggregate level.

2: Diffusion Models: A Background

New products (in particular really new products) undergo a diffusion process: From an initial stage (in which there are zero buyers) individuals start to adopt the innovation and buy the product until the relevant market completely adopts it. Diffusion models try to explain and predict diffusion rates as a function of type of innovation, communication channels, nature of the social systems etc. Despite the large number of factors the models are parsimonious. The history of diffusion research in marketing is briefly presented below:

a) 1969: The Bass model

The modeling of the aggregate penetration of new products in the marketing literature generally follows the Bass model [7]. The model follows Rogers' diffusion of innovations theory of 1962 [1] which emphasizes the role of communication methods: external influence (e.g., advertising, mass media) and internal influence (e.g. WOM = Word Of Mouth), as driving the product adoption pattern. Thus, an individual's probability of adopting a new product at time t (given that s/he had not adopted yet) depends in the Bass model linearly on two forces: a force which is not related to previous adopters and is represented by the parameter of external influence (traditionally denoted as p), and a force that is related to the number of previous adopters, the parameter of internal influence (denoted as q). The hazard model that describes the conditional probability of adoption at time t is:

$$f(t)/[1 - F(t)] = p + qF(t) \tag{1}$$

where $f(t)$ is the probability of adoption at time t and $F(t)$ describes the cumulative probability of adoption. Generally, p represents the effect of external influences, i.e., influence not related to the number of previous adopters, such as advertising. q represents the effect of internal influence, coming from previous adopters.

In the marketing practice eq.(1) is used in the form of eq.(2) in which $n(t)$ represents the buyers (or adopters) within a specified time interval and $N(t)$ is the cumulative number of buyers in a market of M possible buyers:

$$n(t) = [p + q(N(t)/M)][M - N(t)] \tag{2}$$

The Bass model has four main properties: i) It is the most dominant and popular. ii) It fits well many data. iii) After enough data points it is used in practice to forecast sales. iv) However its relevance to a real consumer behavior is questioned in several papers. Its significance (at least to the marketing people) lies also in the fact that the two main parameters can represent internal effects (due to previous adopting population) and external effects (not related to previous adopting population). In that it follows the

diffusion of innovations theory, one of the well known theories of social sciences, that attributes the adoption rate of innovations to communication processes such as Word of Mouth from previous adopters (an internal effect) and mass media influence (an external effect).

b) 1978-79: Extensions of the basic Bass models

Modifications to increase the precision of the model in various cases were suggested. As an example consider a new class of flexible diffusion models, which allow non-symmetric patterns, heterogeneous adopters population etc. Those modifications were motivated by the need for better fit to real life data. A typical model from this generation is:

$$n(t) = [a + b(N(t)/c(M(t))^{1+d}][cM(t) - N(t)]^{1+e} \qquad (3)$$

where M is the market potential and A, b, c, d, e are estimated from the data.

c) 80's and 90's : more growth models

During the 80's data on product penetration and diffusion were accumulated and diverse patterns were observed leading to suggestion of models with different penetration curves. Since growth modeling is an important occupation in a lot of fields, the marketing literature benefits from other fields' achievements. But the main occupation consisted of tailoring a Bass-type model to a specific segment of innovation adoption. For example eq. 4 below was found to fit well adoptions of durables in the agricultural context.

$$n(t) = b[N(t)/M(t)] \, [\ln M/n(t)] \qquad (4)$$

In many cases these models do not relate to diffusion theory, rather they offer smoothing of a noisy data better then other regression technique. Furthermore, their relevance to marketing is sometimes criticized [8] because they have little direct marketing application.

3. Shortcomings in this approach

Indeed, this research stream produced many extensions incorporating assumption regarding issues such as the effect of marketing mix, competition, repeat purchase and technological substitution (see, for example, reviews [3,8,11]). The prediction ability was reported to be satisfactory for various practical implications. However, it seems that this aggregate modeling approach reaches its limits. For instance, how can the coefficients of the smoothing function be interpreted in the individual level? This does not come straight-forward from the Bass model equation, where p and q are part of the linear combination that governs the hazard rate.

Aggregate diffusion models make very simplifying assumptions that assume homogeneity in the communication behavior of adopters. However, while concern regarding this issue has been expressed throughout the diffusion literature (e.g. [12]), because of the nature of the very aggregate data available to researchers, limited options were available to those who wanted to examine these assumptions, and their implications.

In this paper we demonstrate how a microscopic presentation (more precisely percolation modeling) can be used to link market level models to individual level behavior. Further, it will allow us to examine the effect of heterogeneity in the communication behavior of adopters on the aggregate adoption level that are typically analyzed in aggregate

diffusion models such as the Bass model and its extensions [3]. In short, we replace the prevailing mean field theory by a more microscopic statistical approach, taking into account fluctuations and spatial correlations.

4. The percolation representation of product diffusion

Our technique is at once simple, direct and very powerful: represent in the computer the individual buyers, products and sales as well as the information transfer, and the changes in their current individual status.

Each site i of a large lattice is occupied with a random number p_i between zero and one, representing the customer's quality expectation. The quality of a new product is called Q, and potential customers buy it only if this quality is above their expectations: $Q > p_i$ [13]. This standard percolation model [14,15] has a critical percolation threshold p_c such that for $Q > p_c$ an infinite cluster of neighboring buyers can be formed, while for $Q < p_c$ all clusters of buyers are finite. There is a formal equivalence between this picture and the marketing phenomena: far below a certain quality level the product does not sell at all, while far above that density of buyers, the product reaches most of its potential market. The percolation literature [14,15] contains much information about the spatial geometry of clusters which could be used for market modelling.

As long as the p_i do not change, the cluster structures for different Q are correlated. This suggests that one can use the recorded dynamics of one sweep in order to predict the behavior of the subsequent ones, or in general in order to characterize the cluster structure of the market. This line of thought is natural in the context of microscopic simulation but is quite novel in marketing.

Of course in reality even a product which "makes it" may produce losses if the producer over estimates its market share and keeps producing after this is exhausted. On the other hand, the fluctuations (which the percolation model predicts) may discourage a producer and lead him to discontinue the production (flop) even in conditions in which the product could "make it".

In addition to the basic capability to express detailed spatio-temporal knowledge on the market structure and behavior the model above introduces significant conceptual departures from the main features and assumptions of the Bass model.

a) In the Bass model, the fluctuations around the Bass formula are assigned to measurements errors or to repeat purchases (especially close to the peak) in the microscopic simulation the fluctuations are the result of the random irregularities in the connectivity between various parts of the system. In fact fluctuations in the sales rate can appear even if one excludes the possibility of repeated purchases. In particular one can identify strongly connected clusters within which the sales front advances fast separated by regions poor in potential buyers which correspond to stagnation or slowing down in the sales.

b) We could generalize this site percolation picture to a site-bond percolation model, where connections between neighboring buyers are formed only with some bond probability; and then these bond probabilities increase with time if neighboring sites have bought a product. Then, if one allows for the existence of successive product waves (annual issues of the car models, movies in a series like star wars, pink panther, etc.) one obtains smoother curves and larger clusters than in the initial wave. This is due to the emergence of "battered

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paths": herds of buyers with regularly coordinated coherent response to the product. One can say that once the connections between the buyers clusters are established, they are less effective in slowing the propagation of the product sales front.

c) The functional form of the Bass curve is affected too: rather than an exponential increase during the entire pre-saturation region, one reaches a linear (or in general power) region of sales increase once a clear propagation front is formed. Indeed, the surface of a d-dimensional ball increases as the radius to the power $d - 1$.

d) The products which take off are the ones which happen to be planted in a cluster rich in potential buyers. If the cluster is small, the product sales will halt upon reaching the cluster boundary. If the cluster is large, the sales will be much higher.

e) Note that the usual Bass model is just the exact solution of the model in the extreme case where the "neighbors" which link to each site are chosen randomly on the lattice (then the effect of common neighbors is negligible as long as the finite size is negligible) and in which instead of having a buy or a refuse to buy at each site one has always a buy (possibly with a different quantity expressing the buyer preference). In that case, which ignores the discrete character of the buying event (and the discrete choices of the discrete buyers) one gets an exponential increase followed by saturation and one has a perfect averaging of the buying rate by the various quantities bought by the buyers at the buying front.

f) In the case when the product quality Q and the quality expectation p_i change in time [13], the adaptation of the buyers tastes and the producers offer (in terms of quality and price) has the effect that after a few waves of similar products the system will be roughly [17] at the boundary between the exponential decay and exponential increase of sales. Moreover one observes a certain convergence of tastes of the buyers towards a common behavior or towards separate groups which have convergent behavior within the group and divergent between the groups (large regions which react to a new product in a series coherently within the group and disjoint among the groups).

g) On top of this one can consider modeling the effects of peer pressure: sites which are not potential buyers becoming buyers when many of the neighbors bought the product. Sometimes this is not just a psychological effect: it is related with the utility of the product depending on its use by the other buyers (like in the case of fax, ps, pdf, word files formats).

h) The above simple percolation model with time-independent p_i and Q was used to produce in our figure various curves for $n(t)$, the number of new buyers which are neighbors to site which have already bought the product in the previous time interval. We start with one buyer and then let the buying spread over the lattice with a Leath algorithm [16]: At each time step, all neighbors of all previous buyers decide, once and for all, if they buy. Thus our time steps are microsteps in the sense of Huang [17], for one spread of one product through the lattice, and not macrosteps in the sense of Solomon et al [13] referring to repeated attempts with different parameters like Q. We see in fig.1 single examples below, at and above the percolation threshold (no infinite cluster, one fractal infinite cluster, and one compact infinite cluster, respectively). In reality the time resolution may be less fine than in these simulations; this could be taken into account by binning together several consecutive time steps and thus reducing the short-time fluctuations without changing the

long-time trends. Fig.2 shows averages over many samples at the percolation threshold; then the fluctuations vanish. Fig.3 shows such results also for higher dimensions d where the initial sales grow roughly as t^{d-1}, approaching for $d \to \infty$ the exponential growth of the traditional theories like eq.1. (For figures 2 and 3 we used the fully dynamic model [13,17] where q and the p_i self-organize towards the percolation threshold p_c in steps of 0.001; for fixed $q = p_c$ and fixed p_i as for fig.1, the slopes are smaller and depend on whether we average over all clusters or only over the "infinite" clusters.) For comparison, Fig.4 shows two examples of real markets, for automobiles and for LCD color television sets in the 20th century, indicating, respectively, an exponential increase (or power law with a large exponent like 5) and a linear growth. The first example may be better described by a Bass-type theory, the second better by two-dimensional percolation as in Fig. 3.

i) As is customary for phase transitions in physics, the percolation transition implies that even if the probability distribution of the p_i across the lattice is totally uniform, one ends up with localized clusters and sub-clusters of all scales including macroscopic inhomogeneity leading to macroscopic sales rate fluctuations. The fractal clustered character of the market and the bottlenecks are not detectable by the usual polling techniques. A uniform customer distribution leads at the percolation threshold to un-passable barriers and to (almost-) extinction of sales. Similarly, a minor increase of temperature can make the water boiling, without any change in the intermolecular forces.

5. Review of Cluster Geometry

We summarize here some well known percolation properties which could become relevant in a future more quantitative theory of marketing along our outlines. The geometry of percolation clusters has been studied since decades [14,15]. Their surface should not be defined as the set of empty neighbors of occupied sites since the number of such empty neighbors is proportional to the number of occupied sites. Instead, the fractal dimension D is a widespread quantitative measure, defined through $M \propto R^D$ for large clusters with radius R and M occupied sites. On two-dimensional lattices, D is about 1.6, 1.9 and exactly 2 for q below, at and above p_c. For the largest cluster at p_c one can replace the radius R by the linear lattice dimension L in the above mass-radius relation. The fluctuations in the mass of the largest cluster are at $q = p_c$ about as large as the average mass, even if L goes to infinity.

The largest cluster at the percolation threshold, also called the incipient infinite cluster, consists mostly of dangling ends, that means of links through which no current flows if a voltage is applied to two points of the cluster. The current-carrying part of the backbone, varying as $L^{1.6}$, and consists mostly of sites which can be removed without cutting the cluster into parts. Red sites are the bottlenecks, removal of which cuts the cluster into separate parts; their number increases only as $L^{0.75}$. The number ℓ of sites which link within the incipient infinite cluster two sites at Euclidean distance r is called the chemical distance and corresponds to the time (microsteps) needed in our model to transfer information from one customer to the other; it varies on average as $r^{1.1}$ for large distances.

All these exponents are valid rather generally in two dimensions, not only for nearest-neighbor connections on the square lattice. For example, we could allow information to flow also to the four next-nearest neighbors in addition to the four nearest neighbors. Then the value of p_c would change but the above exponents would be the same and thus

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universal. Only if this distance of neighbors goes to infinity do we expect a behavior more similar to eqs.(1-4). Thus these exponents as opposed to $p_c = 0.592746$ are rather general quantitative predictions of percolation theory to be compared with future high-precision data from real marketing.

6. Summary

The traditional approach towards marketing theory, in the literature cited here, has been replaced by a percolation model which treats each customer individually instead of averaging over all of them. As a result strong fluctuations are observed, as found in real sales curves. Just as other percolation applications, also the present one can be modified in numerous ways to describe better specific effects.

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Figure Captions
Fig.1. Examples of simulated sales curves below (line), at (b, +) and above (x) the percolation threshold (part a). Parts b and c show the cluster geometry for the three curves of part a, after the cluster stopped growing or touched the upper boundary.
Fig.2. Averaged sales curves at the percolation threshold for $L \times L$ squares with $L = 100$, 200, 500 and 1001 (from left to right): The individual fluctuations are washed out in the average.
Fig.3. Averaged sales curves for two to five dimensions; the straight lines give the theoretically expected slopes $d - 1$ in this log-log plot.
Fig.4. Yearly sales (in thousands) in the USA of automobiles (a) and of LCD color television sets (b), versus year [18].
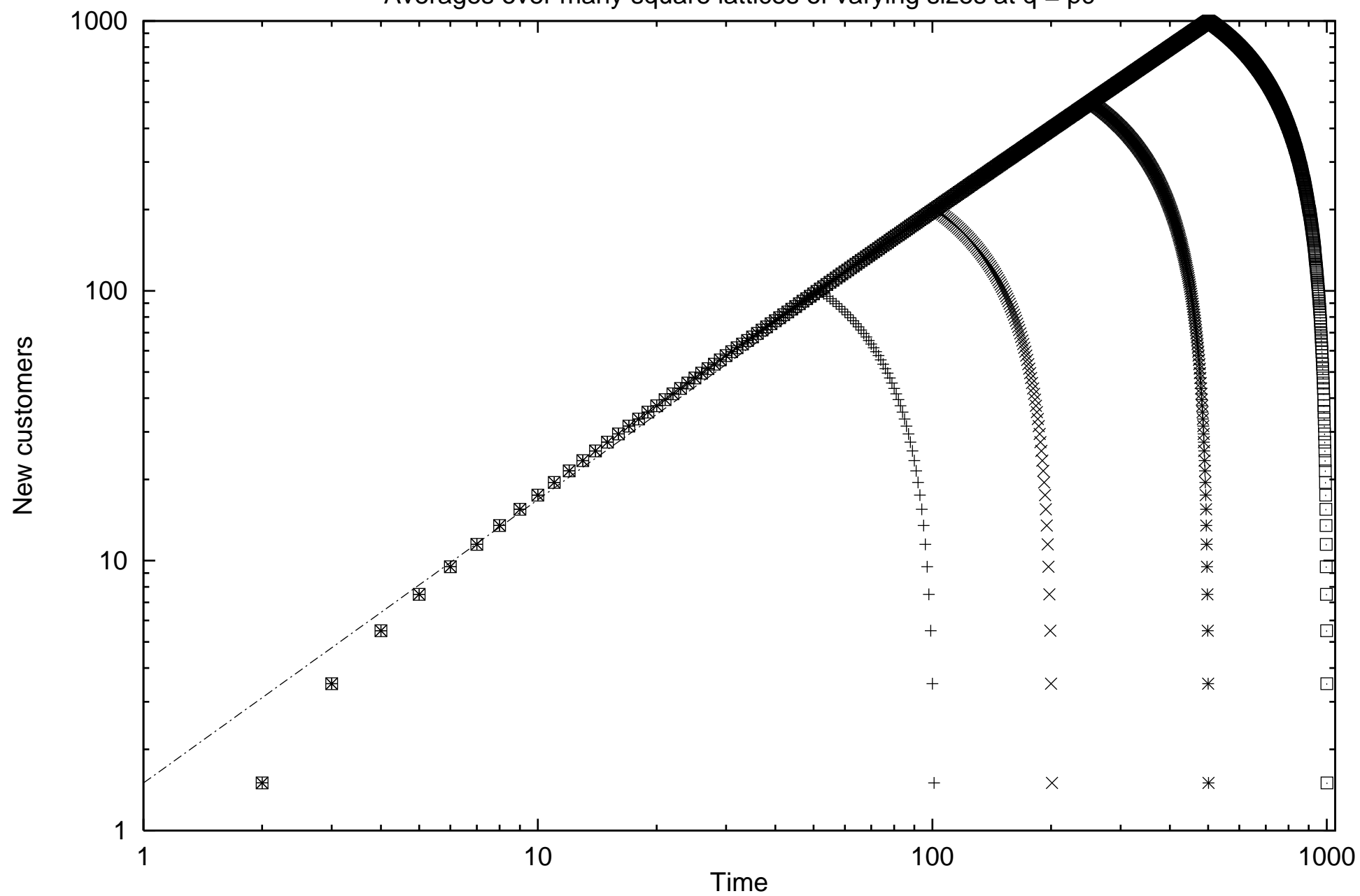
n(t) for 301 * 301 percolation model, q - threshold = -0.05 (line), 0.00 (+), +0.05 (x)
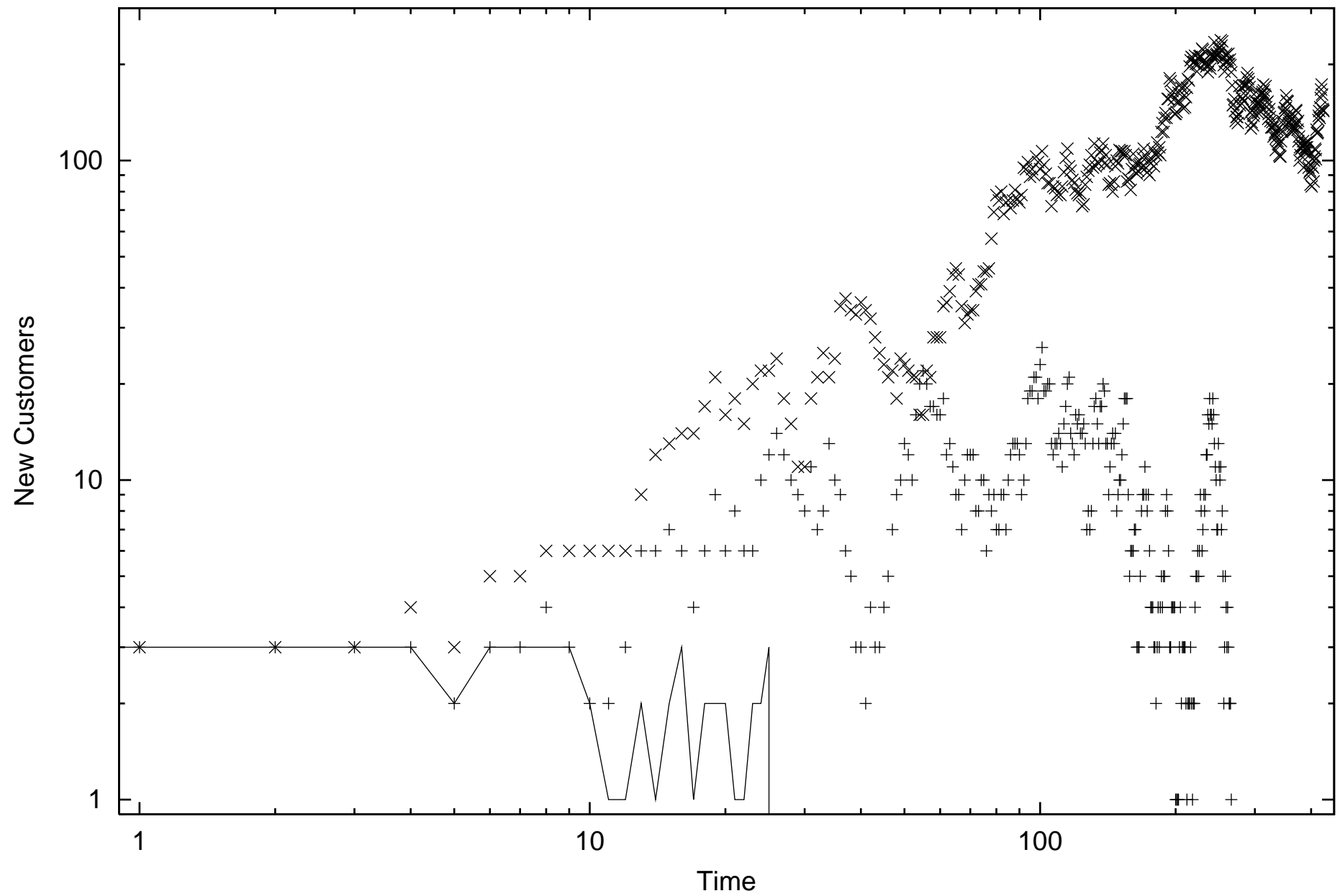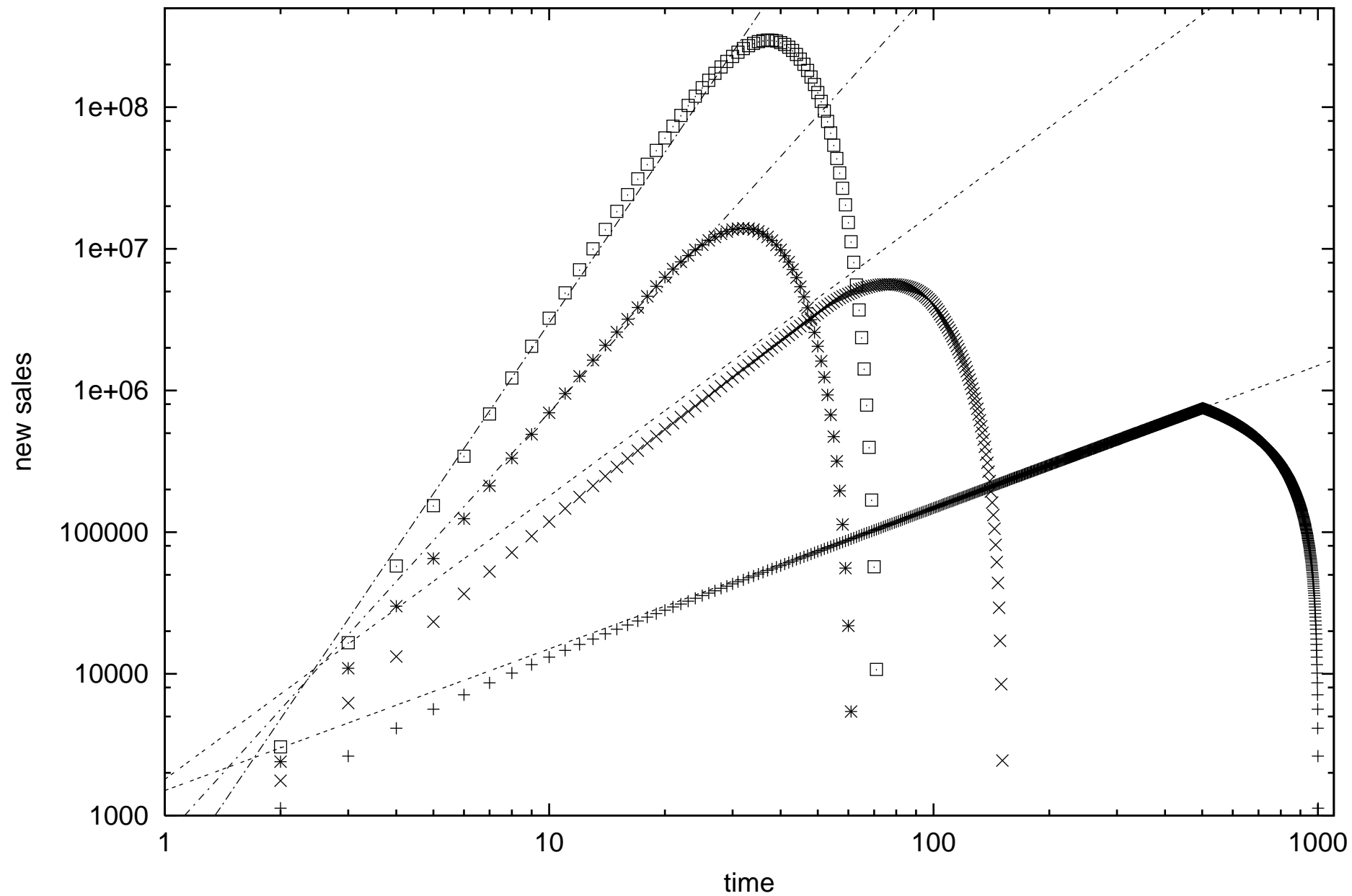
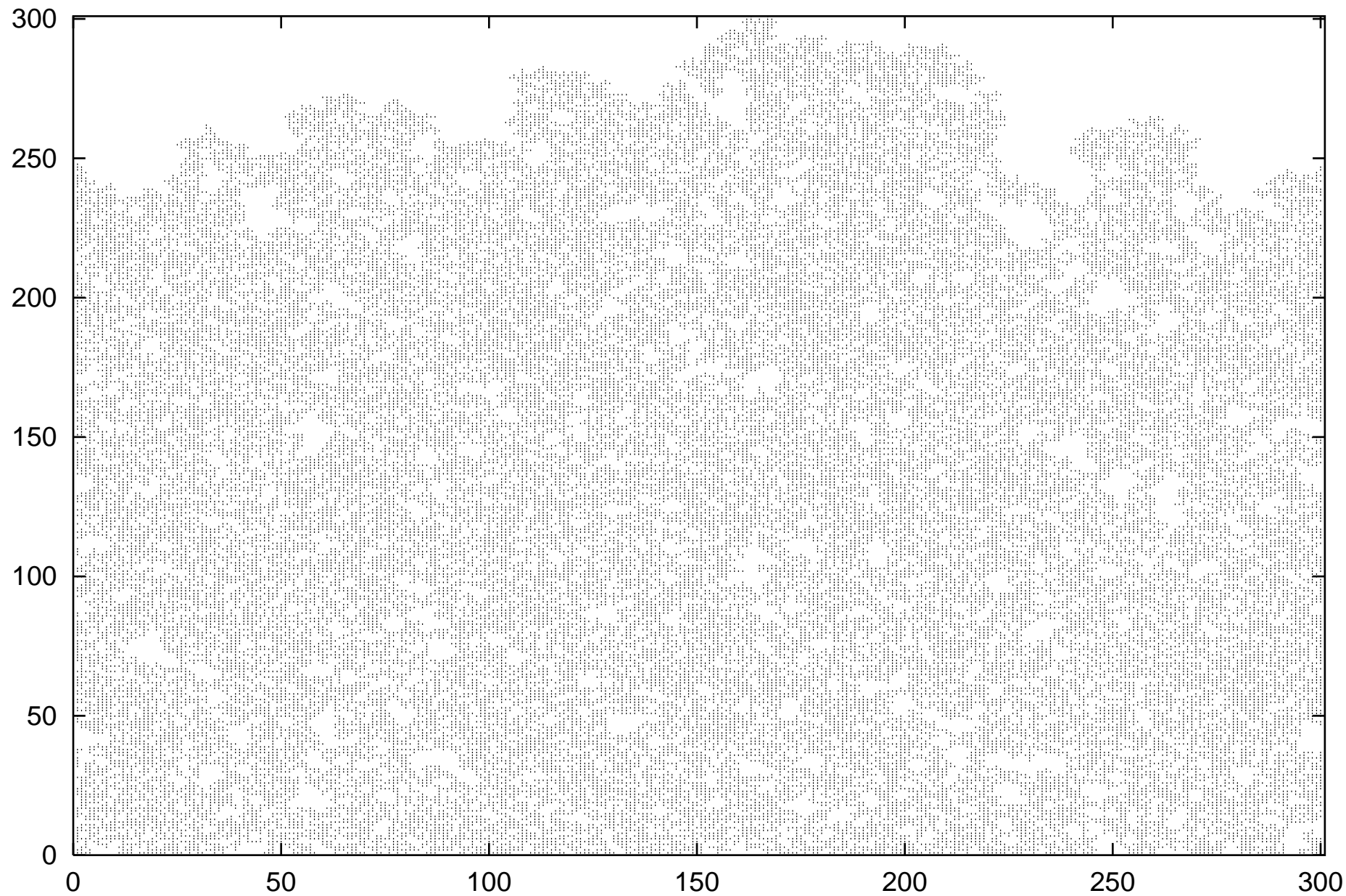Fractal cluster at q=pc and small cluster at q=pc-0.05 in 301*301 lattice
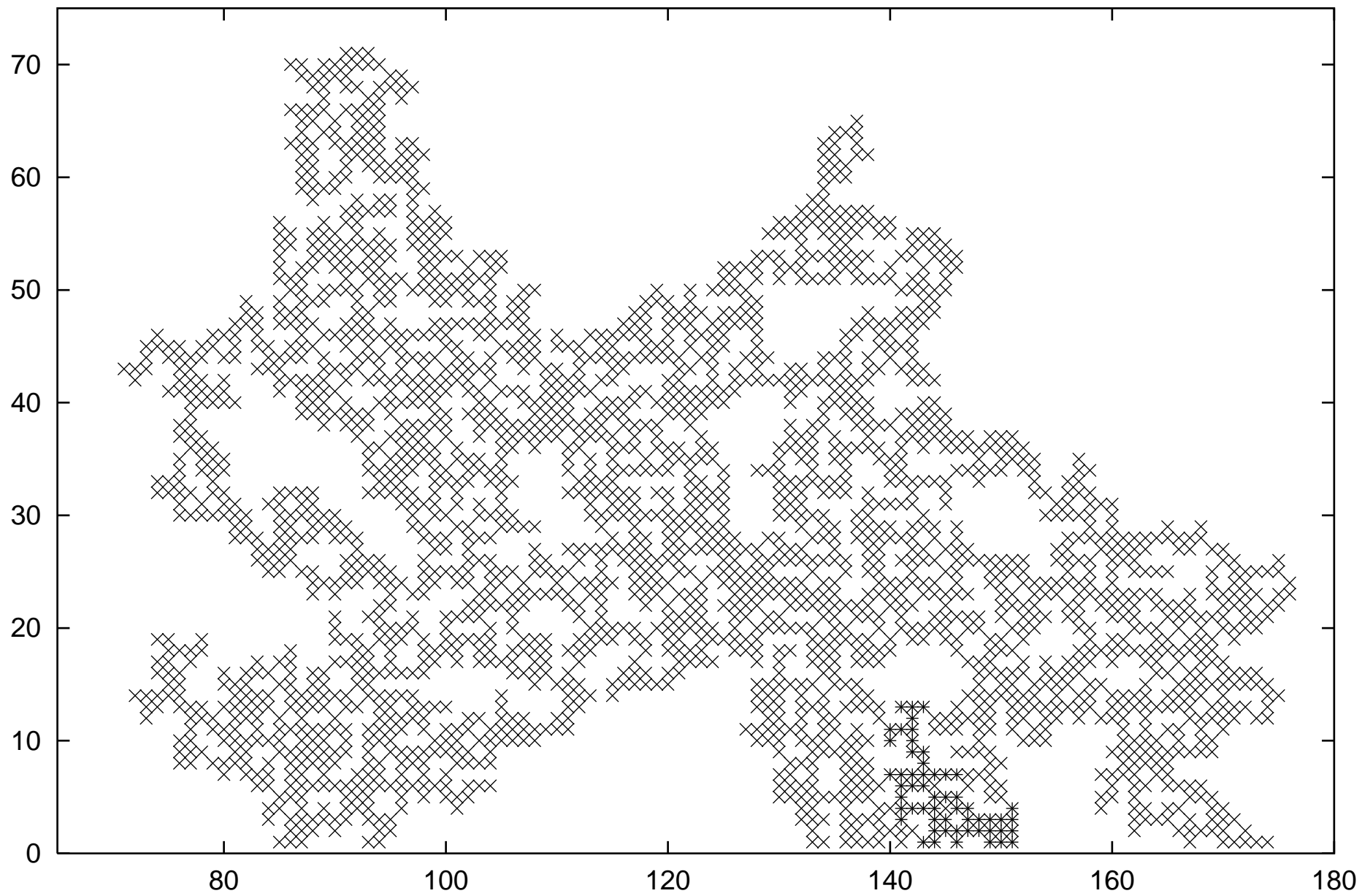
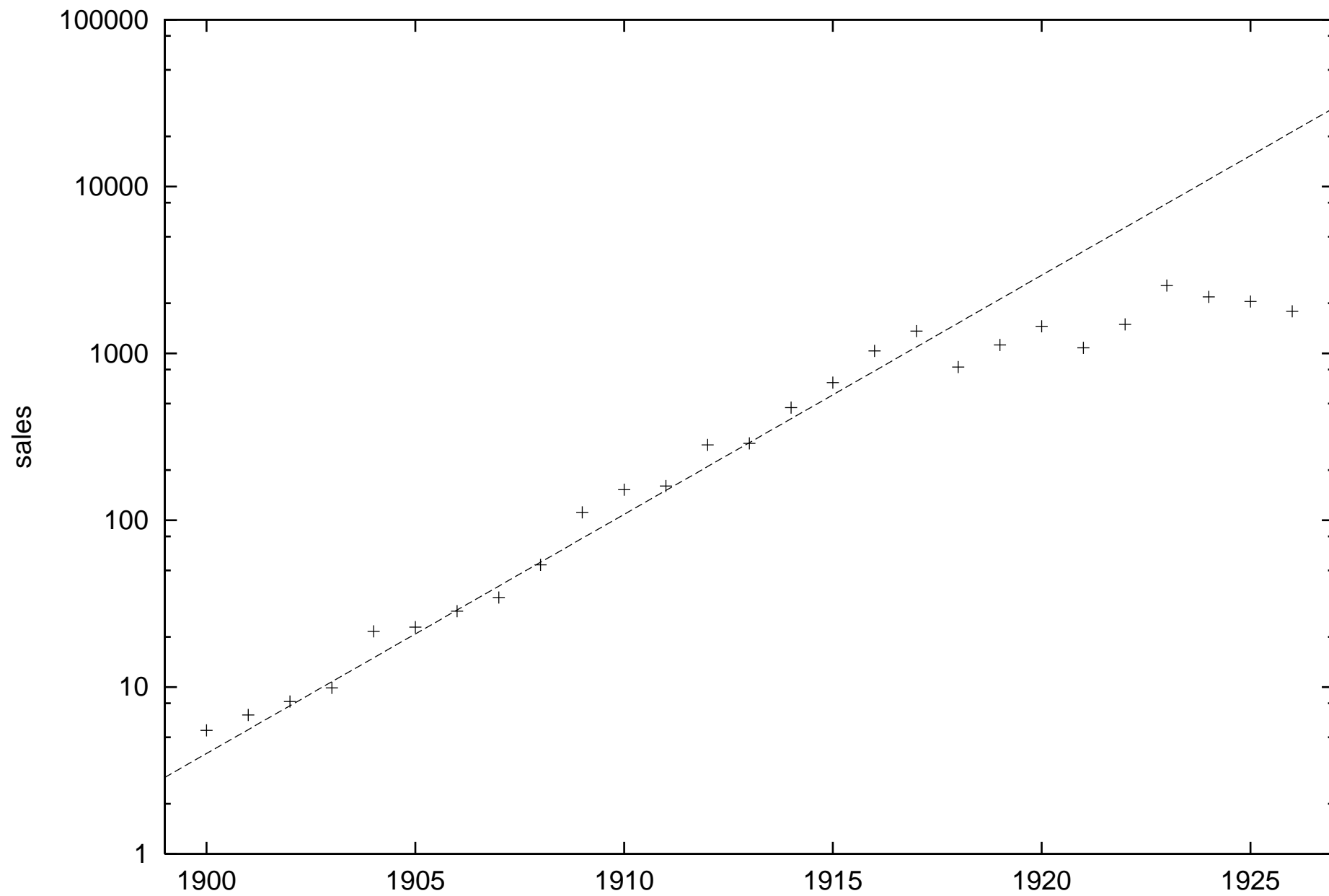Averages over many square lattices of varying sizes at q = pc

n(t) summed over many dynamic lattices 1001^2 (+), 101^3 (x), 31^4 (*) and 29^5 (squares)
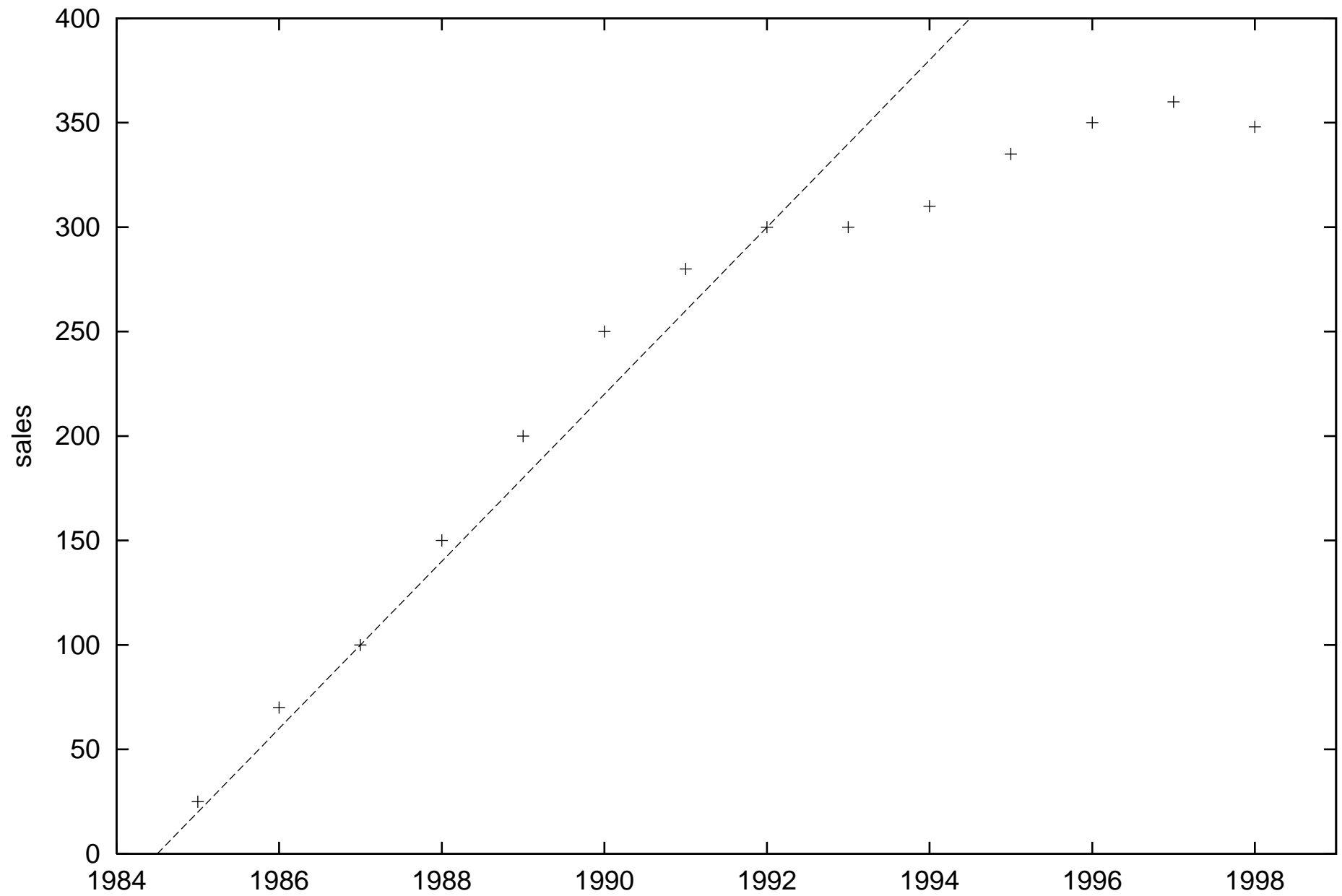
Percolating cluster touching upper boundary of 301*301 lattice, q = pc + 0.05
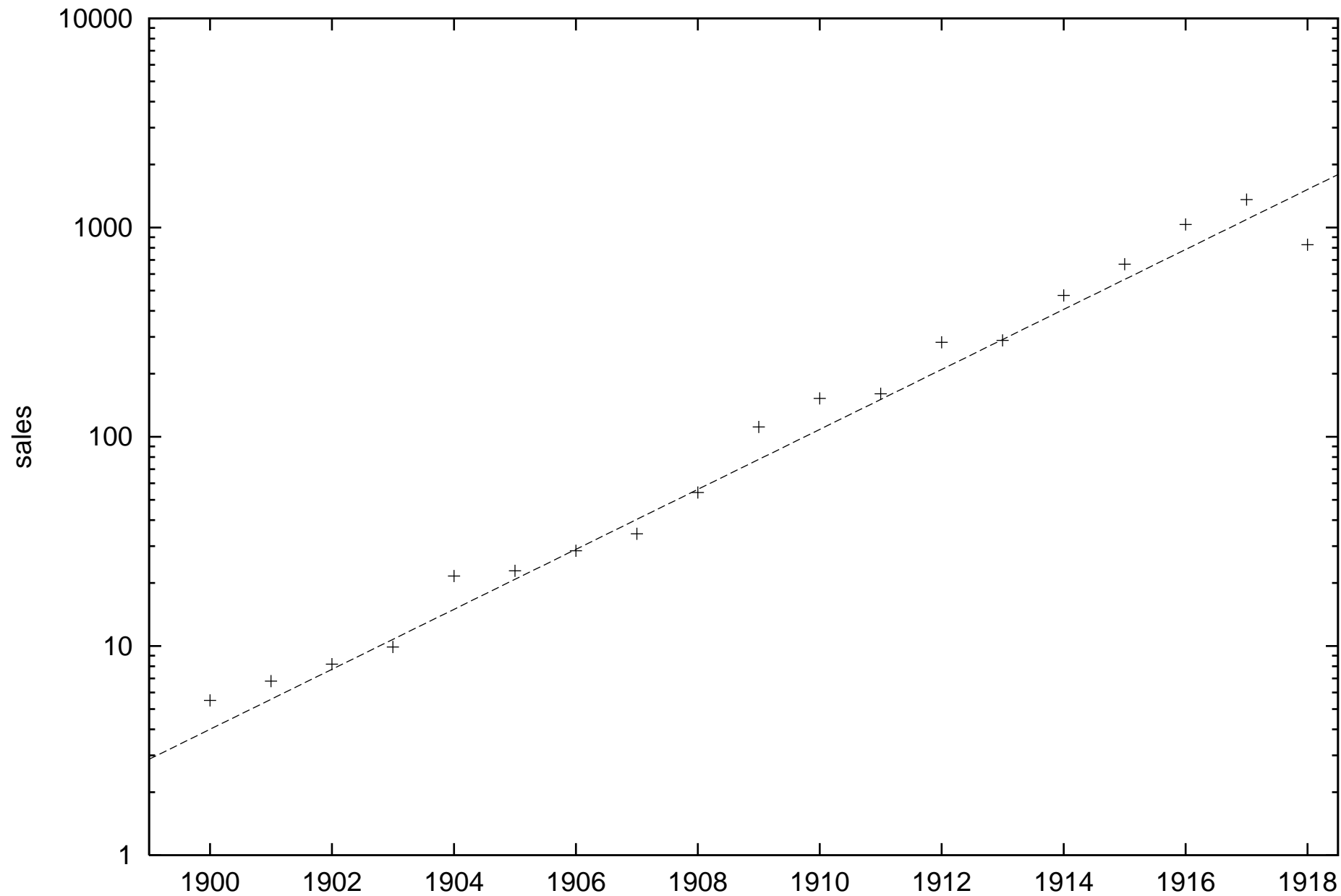
Yearly sales (in thousands) of automobiles in the USA

Yearly sales (in thousands) of LCD color television sets in the USA

Yearly sales of automobiles (in thousands) up to the entry of the USA into World War I

Automobile data plotted as approporiate for a power law; exponent is 5